

Is $\Theta^+(1540)$ a Kaon–Skyrmion Resonance?

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Abstract

We reconsider the relationship between the bound state and the $SU(3)$ rigid rotator approaches to strangeness in the Skyrme model. For non-exotic $S = -1$ baryons the bound state approach matches for small m_K onto the rigid rotator approach, and the bound state mode turns into the rotator zero-mode. However, for small m_K , we find no $S = +1$ kaon bound states or resonances in the spectrum, confirming previous work. This suggests that, at least for large N and small m_K , the exotic state may be an artifact of the rigid rotator approach to the Skyrme model. An $S = +1$ near-threshold state comes into existence only for sufficiently large $SU(3)$ breaking. If such a state exists, then it has the expected quantum numbers of Θ^+ : $I = 0$, $J = \frac{1}{2}$ and positive parity. Other exotic states with $(I = 1, J^P = \frac{3}{2}^+)$, $(I = 1, J^P = \frac{1}{2}^+)$, $(I = 2, J^P = \frac{5}{2}^+)$ and $(I = 2, J^P = \frac{3}{2}^+)$ appear as its $SU(2)$ rotator excitations. As a test of our methods, we also identify a D-wave $S = -1$ near-threshold resonance that, upon $SU(2)$ collective coordinate quantization, reproduces the mass splittings of the observed states $\Lambda(1520)$, $\Sigma(1670)$ and $\Sigma(1775)$ with good accuracy.

1 Introduction

A remarkable recent event in hadronic physics is the discovery of a $S = +1$ baryon (dubbed the Z^+ or Θ^+) with a mass of 1540 MeV and width less than 25 MeV [1]. This discovery was promptly confirmed in [2, 3, 4, 5]. At present the spin, parity and magnetic moment of this state have not been determined; one group, the SAPHIR collaboration [4], found that the isospin of the Θ^+ is zero. Because it appears as a resonance in the system K^+n , the minimal possibility for its quark content is $uudd\bar{s}$ which is manifestly exotic, *i.e.* it cannot be made out of three non-relativistic quarks. Early speculations on this kind of exotic baryons were made in [6, 7]. Remarkably, a state with these quantum numbers appears naturally [8, 9, 10] in the rigid rotator quantization of the three-flavor Skyrme model [11, 12, 13], and detailed predictions for its mass and width were made by Diakonov, Petrov and Polyakov [14]. Their results provided motivation for the experimental searches that led to the discovery of Θ^+ very close to the predicted parameters.¹

The rigid rotator quantization of the Skyrme model that was used in [14] relied on working directly with $N = 3$ (N is the number of colors). Then the model predicts the well-known **8** and **10** of $SU(3)$, followed by an exotic $\overline{\mathbf{10}}$ multiplet whose $S = +1$ member is the Θ^+ . The approach of [14] began by postulating that the established $N(1710)$ and $\Sigma(1880)$ states are members of the anti-decuplet. Then, using group theory techniques, and constraints from a rigid rotator treatment of chiral solitons, they estimated the mass and width of the other states in this multiplet. They predicted that the lowest member of the anti-decuplet has a mass of 1530 MeV and width of about 9 MeV. These results appear to be confirmed strikingly by experiment.²

In this paper we will follow a somewhat different strategy, trying to develop a systematic $1/N$ expansion for Θ^+ . In the two-flavor Skyrme model, the quantum numbers of the low-lying states do not depend on N , as long as it is odd: $I = J = \frac{1}{2}, \frac{3}{2}, \dots$ (I is the isospin and J is the spin). These states are identified with the nucleon and the Δ [25, 26]. The three-flavor case is rather different, since even the lowest $SU(3)$ multiplets depend on $N = 2n + 1$ and become large as $N \rightarrow \infty$. The allowed multiplets must contain states of hypercharge $N/3$, *i.e.* of strangeness $S = 0$. In the notation where $SU(3)$ multiplets are labeled by (p, q) , the lowest multiplets one

¹For other recent theoretical models of Θ^+ , see, e.g. [15, 16, 17, 18, 19, 20].

²Note that the authors of [21, 22, 23, 24] have argued that the experimental data actually indicate an even smaller width.

finds are $(1, n)$ with $J = \frac{1}{2}$ and $(3, n-1)$ with $J = \frac{3}{2}$ [27, 28, 29, 30, 31]. These are the large N analogues of the octet and the decuplet. The rigid rotator mass formula, valid in the limit of unbroken $SU(3)$, is

$$M^{(p,q)} = M_{cl} + \frac{1}{2\Omega}J(J+1) + \frac{1}{2\Phi} \left(C^{(p,q)} - J(J+1) - \frac{N^2}{12} \right), \quad (1.1)$$

where Ω and Φ are moments of inertia, which are of order N . Using the formula for the quadratic Casimir,

$$C^{(p,q)} = \frac{1}{3}[p^2 + q^2 + 3(p+q) + pq], \quad (1.2)$$

one notes [27, 28, 29, 30, 31] that the lowest lying $SU(3)$ multiplets $(2J, n + \frac{1}{2} - J)$, of spin $J = \frac{1}{2}, \frac{3}{2}, \dots$ obey the mass formula

$$M(J) = M_{cl} + \frac{N}{4\Phi} + \frac{1}{2\Omega}J(J+1). \quad (1.3)$$

Exactly the same multiplets appear when we construct baryon states out of N quarks. The splittings among them are of order $1/N$, as is usual for soliton rotation excitations.

The large N analogue of the exotic antidecuplet is $(0, n+2)$ with $J = \frac{1}{2}$. Its splitting from the lowest multiplets is $\frac{N}{4\Phi} + O(1/N)$. The fact that it is of order N^0 raises questions about the validity of the rigid rotator approach to these states [32, 29, 30]. Indeed, we will argue that a better approximation to these states is provided by the bound state approach [33]. In the bound state approach one departs from the rigid rotator ansatz, and adopts more general kaon fluctuation profiles. This has $O(N^0)$ effect on energies of states even in the low-lying non-exotic multiplets, after $SU(3)$ breaking is turned on. In the limit $m_K \rightarrow 0$ the bound state description of non-exotic baryons smoothly approaches the rigid rotator description, and the bound state wave function approaches the zero-mode $\sin(F(r)/2)$ [33, 34], where $F(r)$ is the radial profile function of the skyrmion.

Our logic leads us to believe that, at least from the point of view of the large N expansion, Θ^+ should be described by a near-threshold kaon-skyrmion resonance or bound state of $S = +1$, rather than by a rotator state (a similar suggestion was made independently in [30, 35]). This is akin to the bound state description of the $S = -1$ baryons in [33] where a possibility of such a description of an exotic $S = +1$ state was mentioned as well. This leads us to a puzzle, however, since, in contrast with the

situation for $S = -1$, for $S = +1$ there is no fluctuation mode that in the $m_K \rightarrow 0$ limit approaches the rigid rotator mode of energy $\frac{N}{4\Phi}$ (this will be shown explicitly in section 3). An essential difficulty is that, for small m_K , this is not a near-threshold state; hence, it is not too surprising that it does not show up in the more general fluctuation analysis. Thus, for large N and small m_K the rigid rotator state with $S = +1$ appears to be an artifact of the rigid rotator approximation (we believe this to be a general statement that does not depend on the details of the chiral lagrangian).

Next we ask what happens as we increase m_K keeping other parameters in the kaon-skyrmion Lagrangian fixed. For $m_K = 495 \text{ MeV}$, and with the standard fit values of f_π and e , neither bound states [34] nor resonances [36] exist for $S = +1$.³ If, however, we increase m_K to $\approx 1 \text{ GeV}$ then a near-threshold state appears. Thus, we reach a surprising conclusion that, at least for large N , *the exotic $S = +1$ state exists only due to the $SU(3)$ breaking and disappears when the breaking is too weak*. While this certainly contradicts the philosophy of [31], it is actually in line with some of the earlier literature (see, for example, [38] and the end of [39]). An intuitive way to see the necessity of the $SU(3)$ breaking for the existence of the exotic is that the breaking keeps it a near-threshold state.

One of the purposes of this paper is to examine how sensitive the existence of this state is to parameter choices. If we set $m_K = 495 \text{ MeV}$, then minor adjustments of f_π and e do not make the $S = +1$ resonance appear. We will find, however, that if the strength of the Wess-Zumino term is reduced by roughly a factor of 0.4 compared to its $SU(3)$ value, then a near-threshold state corresponding to Θ^+ is indeed found. Although we do not have a good a priori explanation for this reduction, it could be caused by unexpectedly large $SU(3)$ breaking effects on this particular term. This issue clearly requires further investigation.

The structure of the paper is as follows. In the next section we will review the rigid rotator approach to Skyrme model at large N , and recall a method of large N expansion introduced in [32], which involves expanding in rigid motions around the $SU(2)$ subgroup of $SU(3)$. In section 3 we proceed to the bound state approach that, by introducing extra degrees of freedom, has $O(N^0)$ effect on baryon spectra. In section 4

³Such states seem to appear in [37]. However, in the normalization of the Wess-Zumino term, which is repulsive for $S = +1$ states, a large factor of $e^2 \sim 30$ was apparently omitted there (compare eq. (7) of [37] with [34]). When this factor is reinstated, both bound states and resonances disappear for standard parameter choices, as claimed in [34, 36] and confirmed in this work.

we review the status of $S = -1$ baryons based on kaon-skyrmion bound states, and also study a near-threshold D-wave resonance [36] that, upon $SU(2)$ collective coordinate quantization, reproduces the observed states $\Lambda(1520)$, $\Sigma(1670)$ and $\Sigma(1775)$ with good accuracy. In section 5 we carry out the search for $S = +1$ kaon-skyrmion bound states and resonances. In section 6 we attempt a different fit with a non-zero pion mass. We offer some concluding remarks in section 7.

2 Three-flavor Skyrme model at large N

The Skyrme approach to baryons begins with the Lagrangian [25]

$$L_{\text{Skyrme}} = \frac{f_\pi^2}{16} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2) + \text{Tr}(M(U + U^\dagger - 2)), \quad (2.1)$$

where $U(x^\mu)$ is a matrix in $SU(3)$ and M is proportional to the matrix of quark masses. Later on, it will be convenient to choose units where $ef_\pi = 1$. There is an additional term in the action, called the Wess-Zumino term:

$$S_{\text{WZ}} = -\frac{iN}{240\pi^2} \int d^5x \epsilon^{\mu\nu\alpha\beta\gamma} \text{Tr}(\partial_\mu U U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \partial_\gamma U U^\dagger). \quad (2.2)$$

In the limit of unbroken $SU(3)$ flavor symmetry, its normalization is fixed by anomaly considerations [40].

The Skyrme Lagrangian is a theory of mesons but it describes baryons as well. The simplest baryons in the Skyrme model are the nucleons. Classically, they have no strange quarks, so we may set the kaon fluctuations to zero and consider only the $SU(2)$ -isospin subgroup of $SU(3)$. Skyrme showed that there are topologically stabilized static solutions of hedgehog form:

$$U_0 = U_{\pi,0} = \begin{pmatrix} e^{i\tau \cdot \hat{r} F(r)} & 0 \\ 0 & 1 \end{pmatrix} \quad (2.3)$$

in which the radial profile function $F(r)$ satisfies the boundary conditions $F(0) = \pi$, $F(\infty) = 0$. By substituting the hedgehog ansatz (2.3) into the Skyrme Lagrangian (2.1), and considering the corresponding equations of motion one obtains an equation for $F(r)$ which is straightforward to solve numerically. The non-strange low-lying excitations of this soliton are given by rigid rotations of the pion field $A(t) \in SU(2)$:

$$U(x, t) = A(t) U_0 A^{-1}(t). \quad (2.4)$$

For such an ansatz the Wess-Zumino term does not contribute. By expanding the Lagrangian about U_0 and canonically quantizing the rotations, one finds that the Hamiltonian is

$$H = M_{cl} + \frac{1}{2\Omega} J(J+1), \quad (2.5)$$

where J is the spin and the c-numbers M_{cl} and Ω are complicated integrals of functions of the soliton profile. Numerically, for vanishing pion mass, one finds that

$$M_{cl} \simeq 36.5 \frac{f_\pi}{e}, \quad (2.6)$$

$$\Omega \simeq \frac{107}{e^3 f_\pi}. \quad (2.7)$$

For $N = 2n + 1$, the low-lying quantum numbers are independent of the integer n . The lowest states, with $I = J = \frac{1}{2}$ and $I = J = \frac{3}{2}$, are identified with the nucleon and Δ particles respectively. Since $f_\pi \sim \sqrt{N}$, and $e \sim 1/\sqrt{N}$, the soliton mass is $\sim N$, while the rotational splittings are $\sim 1/N$. Adkins, Nappi and Witten [26] found that they could fit the N and Δ masses with the parameter values $e = 5.45$, $f_\pi = 129$ MeV. In comparison, the physical value of $f_\pi = 186$ MeV.

A generalization of this rigid rotator treatment that produces $SU(3)$ multiplets of baryons is obtained by making the collective coordinate $A(t)$ an element of $SU(3)$. Then the WZ term makes a crucial constraint on allowed multiplets [11, 12, 8, 9, 13]. As discussed in the introduction, large N treatment of this 3-flavor Skyrme model is more subtle than for its 2-flavor counterpart. When $N = 2n + 1$ is large, even the lowest lying $(1, n)$ $SU(3)$ multiplet contains $(n + 1)(n + 3)$ states with strangeness ranging from $S = 0$ to $S = -n - 1$ [29]. When the strange quark mass is turned on, it will introduce a splitting of order N between the lowest and highest strangeness baryons in the same multiplet. Thus, $SU(3)$ is badly broken in the large N limit, no matter how small m_s is [29]. We will find it helpful to think in terms of $SU(2) \times U(1)$ flavor quantum numbers, which do have a smooth large N limit. In other words, we focus on low strangeness members of these multiplets, whose I, J quantum numbers do have a smooth large N limit, and to try identify them with observable baryons.

Since the multiplets contain baryons with up to $\sim N$ strange quarks, the wave functions of baryon with fixed strangeness deviate only an amount $\sim 1/N$ into the strange directions of the collective coordinate space. Thus, to describe them, one may

expand the $SU(3)$ rigid rotator treatment around the $SU(2)$ collective coordinate. The small deviations from $SU(2)$ may be assembled into a complex $SU(2)$ doublet $K(t)$. This method of $1/N$ expansion was implemented in [32], and reviewed in [29].

From the point of view of the Skyrme model the ability to expand in small fluctuations is due to the Wess-Zumino term which acts as a large magnetic field of order N . The method works for arbitrary kaon mass, and has the correct limit as $m_K \rightarrow 0$. To order $O(N^0)$ the Lagrangian has the form [32]

$$L = 4\Phi \dot{K}^\dagger \dot{K} + i\frac{N}{2}(K^\dagger \dot{K} - \dot{K}^\dagger K) - \Gamma K^\dagger K. \quad (2.8)$$

The Hamiltonian may be diagonalized:

$$H = \omega_- a^\dagger a + \omega_+ b^\dagger b + \frac{N}{4\Phi}, \quad (2.9)$$

where

$$\omega_\pm = \frac{N}{8\Phi} \left(\sqrt{1 + (m_K/M_0)^2} \pm 1 \right), \quad M_0^2 = \frac{N^2}{16\Phi\Gamma}. \quad (2.10)$$

The strangeness operator is $S = b^\dagger b - a^\dagger a$. All the non-exotic multiplets contain a^\dagger excitations only. In the $SU(3)$ limit, $\omega_- \rightarrow 0$, but $\omega_+ \rightarrow \frac{N}{4\Phi} \sim N^0$. Thus, the “exoticness” quantum number mentioned in [31] is simply $E = b^\dagger b$ here, and the splitting between multiplets of different “exoticness” is $\frac{N}{4\Phi}$, in agreement with results found from the exact rigid rotator mass formula (1.1).

The rigid rotator prediction for $O(N^0)$ splittings are not exact, however, for reasons explained long ago [33, 32, 29] and reviewed in the introduction. Even for non-exotic states, as the soliton rotates into strange directions, it experiences deformation which grows with m_K . The bound state approach allows it to deform, which has a significant $O(N^0)$ effect on energy levels. We now turn to review the bound state approach.

3 Review of the Bound State Approach

Another approach to strange baryons, which proves to be quite successful in describing the light hyperons, is the so-called bound state method [33]. The basic strategy involved is to expand the action to second order in kaon fluctuations about the classical hedgehog soliton. Then one can obtain a linear differential equation for the kaon field,

incorporating the effect of the kaon mass, which one can solve exactly. The eigenenergies of the kaon field are then precisely the differences between the Skyrmon mass and the strange baryons. In order to implement this strategy, it is convenient to write U in the form

$$U = \sqrt{U_\pi} U_K \sqrt{U_\pi}, \quad (3.1)$$

where $U_\pi = \exp[2i\lambda_j\pi^j/f_\pi]$ and $U_K = \exp[2i\lambda_a K^a/f_\pi]$ with j running from 1 to 3 and a running from 4 to 7.⁴ The λ_a are the standard $SU(3)$ Gell-Mann matrices. We will collect the K^a into a complex isodoublet K :

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} K^4 - iK^5 \\ K^6 - iK^7 \end{pmatrix} = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}. \quad (3.2)$$

Though the Wess-Zumino term can only be written as an action term, if we expand it to second order in K , we obtain an ordinary Lagrangian term:

$$L_{WZ} = \frac{iN}{f_\pi^2} B^\mu \left(K^\dagger D_\mu K - (D_\mu K)^\dagger K \right) \quad (3.3)$$

where

$$D_\mu K = \partial_\mu K + \frac{1}{2} \left(\sqrt{U_\pi}^\dagger \partial_\mu \sqrt{U_\pi} + \sqrt{U_\pi} \partial_\mu \sqrt{U_\pi}^\dagger \right) K, \quad (3.4)$$

and B_μ is the baryon number current. Now we decompose the kaon field into a set of partial waves. Because the background soliton field is invariant under combined spatial and isospin rotations $\mathbf{T} = \mathbf{I} + \mathbf{L}$, a good set of quantum numbers is T, L and T_z , and so we write the kaon eigenmodes as

$$K = k(r, t) Y_{TLT_z}. \quad (3.5)$$

Substituting this expression into $L_{Skyrme} + L_{WZ}$ we obtain an effective Lagrangian for the radial kaon field $k(r, t)$:

$$L = 4\pi \int r^2 dr \left(f(r) \dot{k}^\dagger \dot{k} + i\lambda(r) (k^\dagger \dot{k} - \dot{k}^\dagger k) - h(r) \frac{d}{dr} k^\dagger \frac{d}{dr} k - k^\dagger k (m_K^2 + V_{eff}(r)) \right),$$

⁴There is actually a second coupling constant f_K which replaces f_π in the definition of U_K ; experimentally, $f_K \sim 1.22f_\pi$. To incorporate the difference between these coupling constants, one simply replaces f_π by f_K when expanding in powers of the kaon field, but does not rescale the kinetic and kaon mass terms, which are required to have standard normalization. Then those terms that follow from the four-derivative term, the Wess-Zumino term, and the pion mass in the Skyrme Lagrangian change by a factor of $(f_\pi/f_K)^2$.

with $\lambda(r) = -\frac{Ne^2}{2\pi^2 r^2} F' \sin^2 F$, $f(r) = 1 + 2s(r) + d(r)$, $h(r) = 1 + 2s(r)$, $d(r) = F'^2$, $s(r) = (\sin F/r)^2$, $c(r) = \sin^2 \frac{F}{2}$, and

$$V_{eff} = -\frac{1}{4}(d + 2s) - 2s(s + 2d) + \frac{1 + d + s}{r^2}(L(L + 1) + 2c^2 + 4c\mathbf{I} \cdot \mathbf{L}) \quad (3.6)$$

$$+ \frac{6}{r^2} \left(s(c^2 + 2c\mathbf{I} \cdot \mathbf{L} - \mathbf{I} \cdot \mathbf{L}) + \frac{d}{dr} ((c + \mathbf{I} \cdot \mathbf{L})F' \sin F) \right) - \frac{m_\pi^2}{2}(1 - \cos F).$$

The resulting equation of motion for k is

$$-f(r)\ddot{k} + 2i\lambda(r)\dot{k} + \mathcal{O}k = 0, \quad (3.7)$$

$$\mathcal{O} \equiv \frac{1}{r^2} \partial_r h(r) r^2 \partial_r - m_K^2 - V_{eff}(r).$$

Expanding k in terms of its eigenmodes gives

$$k(r, t) = \sum_{n>0} (\tilde{k}_n(r) e^{i\tilde{\omega}_n t} b_n^\dagger + k_n(r) e^{i\omega_n t} a_n), \quad (3.8)$$

with $\omega_n, \tilde{\omega}_n$ positive. The eigenvalue equations are thus

$$\begin{aligned} (f(r)\omega_n^2 + 2\lambda(r)\omega_n + \mathcal{O})k_n &= 0 & (S = -1), \\ (f(r)\tilde{\omega}_n^2 - 2\lambda(r)\tilde{\omega}_n + \mathcal{O})\tilde{k}_n &= 0 & (S = +1). \end{aligned} \quad (3.9)$$

Crucially, the sign in front of λ , which is the contribution of the WZ term, depends on whether the relevant eigenmodes have positive or negative strangeness. The important result here is the set of equations (3.9) which we will now solve and whose solutions we will match with the spectrum of baryons.

It is possible to examine these equations analytically for $m_K = 0$. The $S = -1$ equation has an exact solution with $\omega = 0$ and $k(r) \sim \sin(F(r)/2)$. This is how the rigid rotator zero mode is recovered in the bound state treatment [34]. As m_K is turned on, this solution turns into an actual bound state [33, 34]. On the other hand, the $S = +1$ equation does *not* have a solution with $\tilde{\omega} = \frac{N}{4\Phi}$ and $k(r) \sim \sin(F(r)/2)$. This is why the exotic rigid rotator state is not reproduced by the more precise bound state approach to strangeness. In section 5 we further check that, for small m_K , there is no resonance that would turn into the rotator state of energy $\frac{N}{4\Phi}$ in the $SU(3)$ limit.

4 Baryons with $S = -1$

In this section we recall the description of $S = -1$ baryons as antikaon–skyrmion bound states or resonances. We will set the kaon mass equal to its physical value, $m_K = 495$ MeV, but set the pion mass equal to zero. If we wish to fit both the nucleon and delta masses to their physical values using the $SU(2)$ rotator approximation, then we must take $e = 5.45$ and $f_\pi = 129$ MeV; let us begin with these values as they are somewhat traditional in analyses based on the Skyrme model.

The lightest strange excitation is in the channel $L=1$, $T = \frac{1}{2}$, and its mass is $M_{cl} + 0.218 ef_\pi \simeq 1019$ MeV. As the lightest state with $S = -1$, it is natural to identify it with the $\Lambda(1115)$, $\Sigma(1190)$, and $\Sigma(1385)$ states, where the additional splitting arises from $SU(2)$ rotator corrections. Let us compute these corrections. The relevant formula for $S = -1$ [34] is

$$M = M_{cl} + \omega_1 + \frac{1}{2\Omega} \left[cJ(J+1) + (1-c)I(I+1) + \frac{3}{4}(c^2 - c) \right], \quad (4.1)$$

where ω_1 is the kaon eigenenergy and c is a number defined in terms of the bound state eigenfunction k_1 by

$$c_{l=1} = 1 - \omega_1 \frac{\int dr k_1^* k_1 \left(\frac{4}{3} f r^2 \cos^2 \frac{F}{2} - 2 \left(\frac{d}{dr} (r^2 F' \sin F) - \frac{4}{3} \sin^2 F \cos^2 \frac{F}{2} \right) \right)}{\int r^2 dr k_1^* k_1 (f\omega_1 + \lambda)}. \quad (4.2)$$

In this $L = 1$, $T = \frac{1}{2}$ channel, we find from numerical integration that $c = 0.617$. The masses, including $SU(2)$ corrections, appear in columns (a) of Table 1. The two features to note here are that first, these states are all somewhat overbound, and second, the $SU(2)$ splittings match rather closely with experiment (one of the successes of the bound state approach [34]).

The next group of strange excitations is in the channel $L = 0$, $T = \frac{1}{2}$, and we have determined its mass before including rotator corrections to be $M_{cl} + 0.523 ef_\pi \simeq 1233$ MeV. In this channel the formula for c is given by

$$c_{l=0} = 1 - \omega_2 \frac{\int dr k_2^* k_2 \left(\frac{4}{3} f r^2 \sin^2 \frac{F}{2} + 2 \left(\frac{d}{dr} (r^2 F' \sin F) + \frac{4}{3} \sin^2 F \sin^2 \frac{F}{2} \right) \right)}{\int r^2 dr k_2^* k_2 (f\omega_2 + \lambda)}, \quad (4.3)$$

and for the relevant bound state this gives $c \sim 0.806$. The $SU(2)$ corrections (4.1) raise the mass of the lightest state in this channel to 1281 MeV. From the quantum

Particle	J	I	L	Mass (expt)	Mass (a)	Mass (b)	Mass (c)
Λ	$\frac{1}{2}$	0	1	1115	1048	1059	1121
Σ	$\frac{1}{2}$	1	1	1190	1122	1143	1289
Σ	$\frac{3}{2}$	1	1	1385	1303	1309	1330
Λ	$\frac{1}{2}$	0	0	1405	1281	1346	1366

Table 1: Masses (in MeV) of the light $S = -1$ hyperons as calculated from the bound state approach, with (a) $e = 5.45$, $f_\pi = f_K = 129$ MeV, (b) $e = 4.82$, $f_\pi = f_K = 186$ MeV, with an overall constant added to fit the N and Δ masses, and (c) the same parameters as (a) but with the WZ term artificially decreased by a factor of 0.4. In all cases $m_\pi = 0$.

numbers, it is natural to identify this state with the $\Lambda(1405)$ state, but as we see it is rather overbound.

We have seen that with the traditional values $e = 5.45$ and $f_\pi = 129$ MeV the Skyrme model successfully captures qualitative features of the baryon spectrum such as the presence of the $\Lambda(1405)$ state, but that the bound states are all too light. It is possible that the zero-point energy of kaon fluctuations, which is hard to calculate explicitly, has to be added to all masses. Thus it is easiest to focus on mass splittings. Then from the $SU(2)$ rotator quantization, we would obtain only one constraint (2.7), from the nucleon- Δ splitting, and be able to adjust e and f_π to improve the fit to known masses. As we increase f_π , we find that the particle masses increase, improving agreement with experiment. For definiteness, let us try to set f_π to its experimental value, $f_\pi=186$ MeV, which then requires $e = 4.82$. We report the results for the masses in column (b) of Table 1.

More dramatic increases in the particle masses may be obtained by distinguishing between the pion decay constant f_π and the kaon decay constant f_K , as shown by Rho, Riska and Scoccola [43], who worked in a modified Skyrme model with explicit vector mesons [44]. For $f_K = 1.22 f_\pi$ they were able to essentially eliminate the over-binding problem for the $L = 1$, $T = \frac{1}{2}$ states, though they still found the analogue of the $\Lambda(1405)$ state to be overbound by about 100 MeV. We should add that the natural appearance of this $\Lambda(1405)$ with negative parity is a major success of the bound state approach [34, 45]. In quark or bag models such a baryon is described by a p -wave quark excitation, which typically turns out to be too heavy (for a discussion, see the

introduction of [45]).

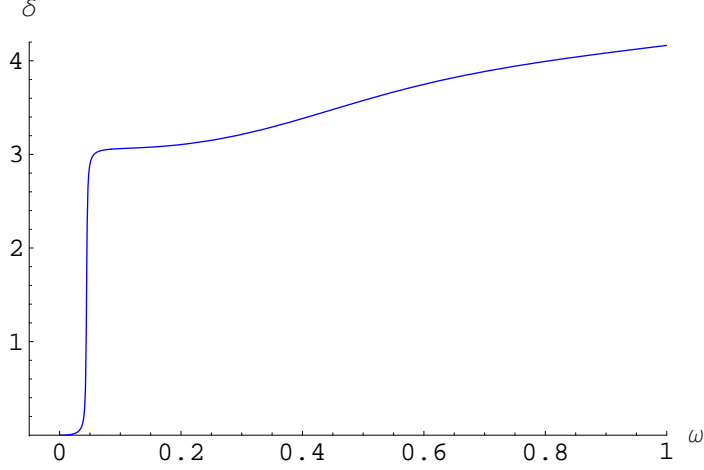


Figure 1: Phase shift as a function of energy in the $L = 2$, $T = \frac{3}{2}$, $S = -1$ channel. The energy ω is measured in units of ef_π (with the kaon mass subtracted, so that $\omega = 0$ at threshold), and the phase shift δ is measured in radians. Here $e = 5.45$ and $f_\pi = 129$ MeV.

A third way to raise the masses of the Λ and Σ states is to modify the Wess-Zumino term. For the $S = -1$ states, the WZ term results in an attractive force between the Skyrmion and the kaon, so if we reduce this term by hand, the hyperons will become less tightly bound and their masses should increase.⁵ We will address this approach in the next section, and we will see that such a reduction helps to produce an $S = +1$ near-threshold state. However, too great a reduction of the WZ term will spoil the hyperfine splittings governed by the parameter c .

Finally, let us note that the philosophy of the bound state approach can be applied successfully to states above threshold. Such states will appear as resonances in kaon-nucleon scattering, which we may identify by the standard procedure of solving the appropriate kaon wave equation and studying the phase shifts of the corresponding solutions as a function of the kaon energy. In the $L = 2$, $T = \frac{3}{2}$ channel there is a resonance at $M_{cl} + 0.7484 ef_\pi = 1392$ MeV (see Figure 1)⁶. In this channel, the $SU(2)$

⁵Note that for unbroken $SU(3)$ the WZ term is quantized and cannot be changed by hand. However, $SU(3)$ breaking is likely to change the WZ term.

⁶The existence of this resonance was noted long ago in a vector-meson stabilized Skyrme model [36].

Particle	J	I	L	Mass (expt)	Mass (th)
Λ (D_{03})	$\frac{3}{2}$	0	2	1520	1462
Σ (D_{13})	$\frac{3}{2}$	1	2	1670	1613
Σ (D_{15})	$\frac{5}{2}$	1	2	1775	1723

Table 2: Masses (in MeV) of the $S = -1$ D -wave resonances calculated from the bound state approach, with $f_\pi = 129$ MeV, $e = 5.45$.

splitting parameter c is given by the formula [41]

$$c_{l=2} = 1 - \omega_3 \frac{\int dr k_3^* k_3 \left(\frac{2}{3} \left(1 + \frac{4}{5} \cos^2 \frac{F}{2} \right) f r^2 - \frac{4}{5} \left(\frac{d}{dr} (r^2 F' \sin F) + \frac{4}{3} \sin^2 F \sin^2 \frac{F}{2} \right) \right)}{\int r^2 dr k_3^* k_3 (f \omega_3 + \lambda)}.$$

Numerically, we evaluate this coefficient by cutting off the radial integral around the point where $k(r)$ begins to oscillate. We find $c \sim 0.23$. The states are split into the channels with (I, J) given by $(0, \frac{3}{2})$, $(1, \frac{3}{2})$, $(1, \frac{5}{2})$ [36], with masses 1462 MeV, 1613 MeV, and 1723 MeV respectively (see Table 2). We see that these correspond nicely to the known negative parity resonances $\Lambda(1520)$ (which is D_{03} in standard notation), $\Sigma(1670)$ (which is D_{13}) and $\Sigma(1775)$ (which is D_{15}) [42]. As with the bound states, we find that the resonances are somewhat overbound (the overbinding of all states is presumably related to the necessity of adding an overall zero-point energy of kaon fluctuations), but that the mass splittings within this multiplet are accurate to within a few percent. In fact, we find that the ratio

$$\frac{M(1, \frac{5}{2}) - M(0, \frac{3}{2})}{M(1, \frac{3}{2}) - M(0, \frac{3}{2})} \approx 1.73, \quad (4.4)$$

while its empirical value is 1.70.

This very good agreement with the states observed above the $K - N$ threshold is an additional success of the kaon fluctuation approach to strange baryons.

5 A Baryon with $S=+1$?

For states with positive strangeness, the eigenvalue equation for the kaon field is the same except for a change of sign in the contribution of the WZ term. This sign change

makes the WZ term repulsive for states with \bar{s} quarks and introduces a splitting between ordinary and exotic baryons [33]. In fact, with standard values of the parameters (such as those in the previous section) the repulsion is strong enough to remove *all* bound states and resonances with $S = 1$, including the newly-observed Θ^+ . It is natural to ask how much we must modify the Skyrme model to accommodate the pentaquark. The simplest modification we can make is to introduce a coefficient a multiplying the WZ term. Qualitatively, we expect that reducing the WZ term will make the $S = +1$ baryons more bound, while the opposite should happen to the ordinary baryons. Another modification we will attempt is to vary the mass of the kaon; we will find that for sufficiently large kaon mass the Θ^+ becomes stable. In all cases, we have found empirically that raising f_K relative to f_π makes the pentaquark less bound, so for this section we will take $f_K = f_\pi$.

The most likely channel in which we might find an exotic has the quantum numbers $L = 1$, $T = \frac{1}{2}$, as in this case the effective potential is least repulsive near the origin. For $f_\pi = 129, 186$, and 225 MeV, with $e^3 f_\pi$ fixed, we have studied the effect of lowering the WZ term by hand. Interestingly, in all three cases we have to set $a \simeq 0.39$ to have a bound state at threshold. If we raise a slightly, this bound state moves above the threshold, but does not survive far above threshold; it ceases to be a sharp state for $a \simeq 0.46$. We have plotted phase shifts for various values of a in Figure 2.⁷ With $a = 0.39$ and $f_\pi = 129$ MeV, this state (with mass essentially at threshold) has $SU(2)$ splitting parameter $c \sim -0.48$. The $SU(2)$ collective coordinate quantization of the state proceeds analogously to that of the $S = -1$ bound states, and the mass formula is again of the form (4.1). Thus the lightest $S = +1$ state we find has $I = 0$, $J = \frac{1}{2}$ and positive parity, *i.e.* it is an $S = +1$ counterpart of the Λ . This is our candidate Θ^+ state. Its first $SU(2)$ rotator excitations have $I = 1$, $J^P = \frac{3}{2}^+$ and $I = 1$, $J^P = \frac{1}{2}^+$ (a relation of these states to Θ^+ also follows from general large N relations among baryons [35]). The counterparts of these $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ states in the rigid rotator quantization lie in the **27**-plets of $SU(3)$. These states were recently discussed in [46, 47].

From the mass formula (4.1) we deduce that

$$M(1, \frac{1}{2}) - M(0, \frac{1}{2}) = \frac{1}{\Omega}(1 - c) , \quad M(1, \frac{3}{2}) - M(0, \frac{1}{2}) = \frac{1}{\Omega} \left(1 + \frac{c}{2}\right) . \quad (5.5)$$

⁷When the state is above the threshold, we do not find a full π variation of the phase. Furthermore, the variation and slope of the phase shift decrease rapidly as the state moves higher, so it gets too broad to be identifiable. So, the state can only exist as a bound state or a near-threshold state.

Since $c < 0$, the $J = \frac{3}{2}$ state is lighter than $J = \frac{1}{2}$. Using $c \sim -0.48$, we find that the $I = 1$, $J^P = \frac{3}{2}^+$ state is ~ 148 MeV heavier than the Θ^+ ,⁸ while the $I = 1$, $J^P = \frac{1}{2}^+$ state is ~ 289 MeV heavier than the Θ^+ .

We may further consider $I = 2$ rotator excitations which have $J^P = \frac{3}{2}^+, \frac{5}{2}^+$. Such states are allowed for $N = 3$ (in the quark language the charge +3 state, for example, is given by $uuu\bar{s}$). The counterparts of these $J^P = \frac{3}{2}^+, \frac{5}{2}^+$ states in the rigid rotator quantization lie in the **35**-plets of $SU(3)$ [46, 47]. From the mass formula (4.1) we deduce that

$$\begin{aligned} M(2, \frac{5}{2}) - M(0, \frac{1}{2}) &= \frac{1}{\Omega}(3 + c) \sim 494 \text{ MeV} , \\ M(2, \frac{3}{2}) - M(0, \frac{1}{2}) &= \frac{3}{\Omega} \left(1 - \frac{c}{2}\right) \sim 729 \text{ MeV} . \end{aligned} \quad (5.6)$$

While the value of c certainly depends on the details of the chiral lagrangian, we may form certain combinations of masses of the exotics from which it cancels. In this way we find “model-independent relations” which rely only the existence of the $SU(2)$ collective coordinate:

$$\begin{aligned} 2M(1, \frac{3}{2}) + M(1, \frac{1}{2}) - 3M(0, \frac{1}{2}) &= 2(M_\Delta - M_N) = 586 \text{ MeV} , \\ \frac{3}{2}M(2, \frac{5}{2}) + M(2, \frac{3}{2}) - \frac{5}{2}M(0, \frac{1}{2}) &= 5(M_\Delta - M_N) = 1465 \text{ MeV} , \\ M(2, \frac{3}{2}) - M(2, \frac{5}{2}) &= \frac{5}{3} \left(M(1, \frac{1}{2}) - M(1, \frac{3}{2}) \right) , \end{aligned} \quad (5.7)$$

where we used $M_\Delta - M_N = \frac{3}{2\Omega}$. These relations are analogous to the sum rule [33]

$$2M_{\Sigma^*} + M_\Sigma - 3M_\Lambda = 2(M_\Delta - M_N) , \quad (5.8)$$

which is obeyed with good accuracy. If the $I = 1, 2$ exotic baryons are discovered, it will be very interesting to compare the relations (5.7) with experiment.

Suppose we tried to require the existence of a pentaquark state by setting $a=0.4$. How would this change affect the spectrum of negative strangeness baryons? The results are somewhat mixed. Let us take $f_\pi=129$ MeV as an example. In the $L = 1, T = \frac{1}{2}$ channel, the bound state has a mass of 1209 MeV before including $SU(2)$ rotator corrections. The parameter c which characterizes the $SU(2)$ splittings falls to

⁸This value is close to those predicted in [46] but is significantly higher than the 55 MeV reported in [47]. For a comparison of these possibilities with available data, see [19].

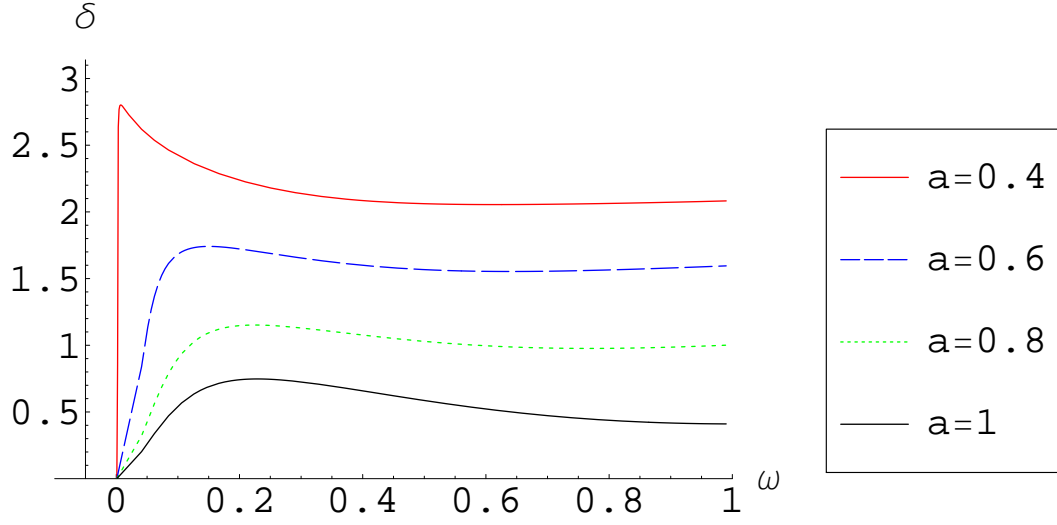


Figure 2: Phase shifts δ as a function of energy in the $S = +1, L = 1, T = \frac{1}{2}$ channel, for various choices of the parameter a (strength of the WZ term). The energy ω is measured in units of ef_π ($e = 5.45$, $f_\pi = f_K = 129$ MeV) and the phase shift δ is measured in radians. $\omega = 0$ corresponds to the $K - N$ threshold.

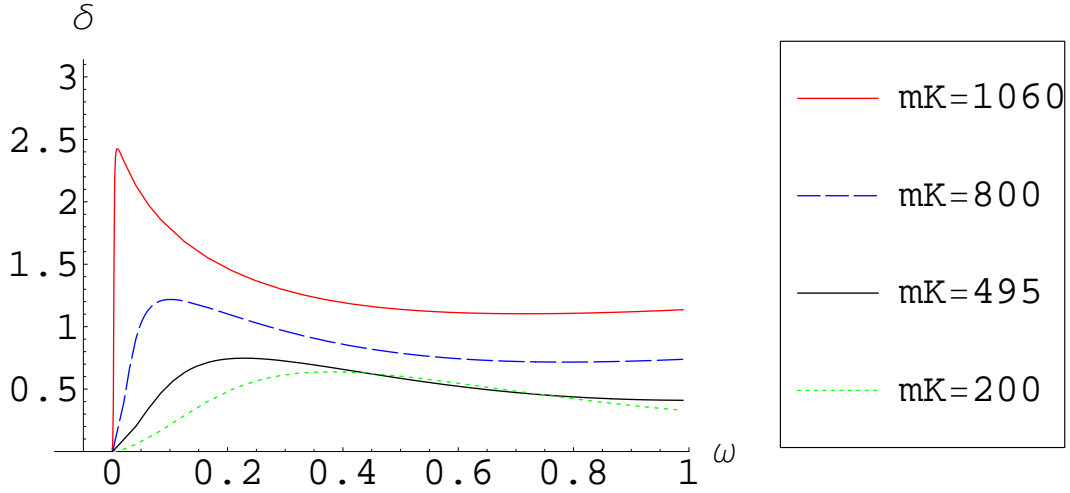


Figure 3: Phase shifts δ as a function of energy in the $S = +1, L = 1, T = \frac{1}{2}$ channel, for various values of m_K . Here $e = 5.45$ and $f_\pi = f_K = 129$ MeV.

$c \sim 0.14$. Including the splittings we find the masses given in column (c) of Table 1. Notice that the Σ is far above its experimental mass of 1190 MeV, signaling drastic disagreement with the Gell-Mann-Okubo relations, as we would expect for this small value of c . In the $L = 0, T = \frac{1}{2}$ channel the Λ resonance is at 1366 MeV (including rotator corrections), still somewhat overbound.

As another probe of the parameter space of our Skyrme model, let us vary the mass of the kaon and see how this affects the pentaquark. As observed in Section 3, in the limit of infinitesimal kaon mass, there is no resonance in the $S = +1, L = 1, T = \frac{1}{2}$ channel. We find that to obtain a bound state in this channel, we must raise m_K to about 1100 MeV.⁹ Plots of the phase shift vs. energy for different values of m_K are presented in Figure 3. This figure clearly shows that increasing $SU(3)$ breaking leads to increasing variation of the scattering phase.

6 Fits with Massive Pion

One interesting way to extend the model is to include the mass of the pion, as first explored by Adkins and Nappi [49]. We find that this actually improves matters for the pentaquark. The basic procedure is simply to take $m_\pi = 138$ MeV; there will be a modification in the kaon effective potential and also a change in the variational equation for the Skyrmion profile function $F(r)$. As a result, the constraints on e and f_π change:

$$M_{cl} \simeq 38.7 \frac{f_\pi}{e}, \quad (6.9)$$

$$\Omega \simeq \frac{62.9}{e^3 f_\pi}. \quad (6.10)$$

Adkins and Nappi found that the best fit to the nucleon and delta masses was given by $e = 4.84$ and $f_\pi = 108$ MeV. In dimensionless units, $m_\pi = .263$.

For the $S = -1$ baryons, we find a bound state in the $L = 1, T = \frac{1}{2}$ channel with mass 1012 MeV. The $SU(2)$ rotator parameter is $c_{l=1} \sim 0.51$, giving a mass spectrum

⁹Since both D and B mesons are much heavier than this, we may infer following [38] that there exist exotic bound state baryons which, in the quark model language, are pentaquarks containing an anti-charm or an anti-bottom quark (provided that the associated meson decay constants f_D and f_B are not too large). This prediction is rather insensitive to the details of the chiral lagrangian [38]. See also [48] for a more careful analysis of these exotics that incorporates heavy quark symmetry.

Particle	J	I	L	Mass (expt)	Mass (d)	Mass (e)
Λ	$\frac{1}{2}$	0	1	1115	1031	1050
Σ	$\frac{1}{2}$	1	1	1190	1126	1177
Σ	$\frac{3}{2}$	1	1	1385	1276	1279
Λ	$\frac{1}{2}$	0	0	1405	1253	1283

Table 3: Masses (in MeV) of the light $S = -1$ hyperons as calculated from the bound state approach, with $e = 4.84$, $f_\pi = f_K = 108$ MeV, and $m_\pi = 138$ MeV. Column (d) reports the masses with the usual WZ term; in column (e) the WZ term has been artificially reduced by a factor 0.75.

of 1031, 1126, and 1276 MeV. In the $L = 0$, $T = \frac{1}{2}$ sector there is a bound state with mass 1204 MeV which is increased by rotator corrections ($c \sim 0.82$) to give a state with mass 1253 MeV corresponding to $\Lambda(1405)$. Furthermore, in the $L = 2$, $T = \frac{3}{2}$ channel there is a bound state slightly below the threshold [41] (in the massless pion fit this state was a resonance slightly above the threshold). While these states are all overbound, one can presumably improve the fit by adjusting parameters as in the previous sections.

Let us first note that with the massive pion fit, the pentaquark appears with a smaller adjustment in parameters. In the $L = 1$, $T = \frac{1}{2}$, $S = +1$ channel, there is a bound state for $a \sim 0.69$. Let us study the effect of setting $a = 0.75$ on the bound states. With this change, the bound state energies for negative strangeness baryons rise. In the $L = 1$, $T = \frac{1}{2}$ channel, the mass is 1114 MeV, and $c_{l=1} \sim 0.35$, giving masses of 1050, 1177 and 1279 MeV for the Λ, Σ, Σ states. $c_{l=1}$ is smaller than its experimental value of 0.62 but is not disastrously small, and the overall masses have increased towards their experimental values. In the $L = 0$, $T = \frac{1}{2}$ channel, the Λ state also increases to 1283 MeV (including rotator corrections.) One further interesting result is that the pentaquark seems more sensitive to the mass of the kaon with $m_\pi = 138$ MeV; the pentaquark actually becomes bound for $m_K = 700$ MeV.

It is also possible to vary the pion mass; for masses around 200 MeV the bound state can appear for $S=1$ with $a \sim 0.8$.

7 Discussion

There are several implications of the analysis carried out in the preceding sections. First of all, we have to admit that the bound state approach to the Skyrme model could not have been used to predict the existence of an exotic $S = +1$ baryon. Indeed, for typical parameter choices we find neither kaon-Skyrmion bound states nor resonances with $S = +1$, confirming earlier results from the 80's [34, 36, 29]. At the time these results appeared consistent with the apparent absence of such exotic resonances in kaon scattering data.¹⁰

We have found, however, that by a relatively large adjustment of parameters in the minimal bound state lagrangian, such as reduction of the WZ term to 0.4 of its $SU(3)$ value, the near-threshold $S = +1$ kaon state can be made to appear. In this case, however, the agreement of the model with the conventional strange baryons is worsened somewhat. A better strategy may be to vary more parameters in the Lagrangian, and perhaps to include other terms; then there is hope that properties of both exotic and conventional baryons will be reproduced nicely. This would be a good project for the future.

Finally, our work sheds new light on connections between the bound state and the rigid rotator approaches to strange baryons. These connections were explored in the 80's, and it was shown that the bound state approach matches nicely to 3-flavor rigid rotator quantization carried out for large N [34, 32, 29]. The key observation is that, in both approaches, the deviations into strange directions become small in the large N limit due to the WZ term acting as a large magnetic field. Thus, for baryons whose strangeness is of order 1, the harmonic approximation is good for any value of m_K . The $S = -1$ bound state mode smoothly turns into the rotator zero-mode in the limit $m_K \rightarrow 0$, which shows explicitly that rotator modes can be found in the small fluctuation analysis around the $SU(2)$ skyrmion. However, for small m_K there is no fluctuation mode corresponding to exotic $S = +1$ rigid rotator excitations. In our opinion, this confirms the seriousness of questions raised about such rotator states for large N [32, 29, 30].

If the large N expansion is valid, then we conclude that the exotic baryon appears in the spectrum only for sufficiently large $SU(3)$ breaking. The simplest way

¹⁰In fact, the experimental situation is still somewhat confused (see [21, 22, 23, 24] for discussions of remaining puzzles).

to parametrize this breaking is to increase m_K while keeping coefficients of all other terms fixed at their $SU(3)$ values. Then we find that the resonance appears at a value of $m_K \sim 1\text{GeV}$. However in reality $SU(3)$ breaking will also affect other coefficients, in particular it may reduce somewhat the strength of the WZ term, thus helping the formation of the resonance. In principle the coefficients in the chiral lagrangian should be fitted from experiment, and also higher derivative terms may need to be included.

What does our work imply about the status of the Θ^+ baryon in the real world? As usual, this is the most difficult question. If $N = 3$ is large enough for the semiclassical approach to skyrmions to be valid, then we believe that our picture of Θ^+ as a kaon-skyrmion near-threshold state is a good one. It is possible, however, that the rigid rotator approach carried out directly for $N = 3$, as in [14], is a better approximation to the real world, as suggested by its successful prediction of the pentaquark. It is also possible that quark model approaches, such as those in [16, 17], or lattice calculations [20], will eventually prove to be more successful. Clearly, further work, both experimental and theoretical, is needed to resolve these issues.

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